Trigonometry

Introduction

you have already studied about triangles, and in particular, right triangles, in earlier classes. Let us take some examples from our surroundings where right triangles can be imagined to be formed.

For instance:

Example I

Suppose the students of a school are visiting Qutub minar. Now, if a student is looking at the top of the minar, a right triangle can

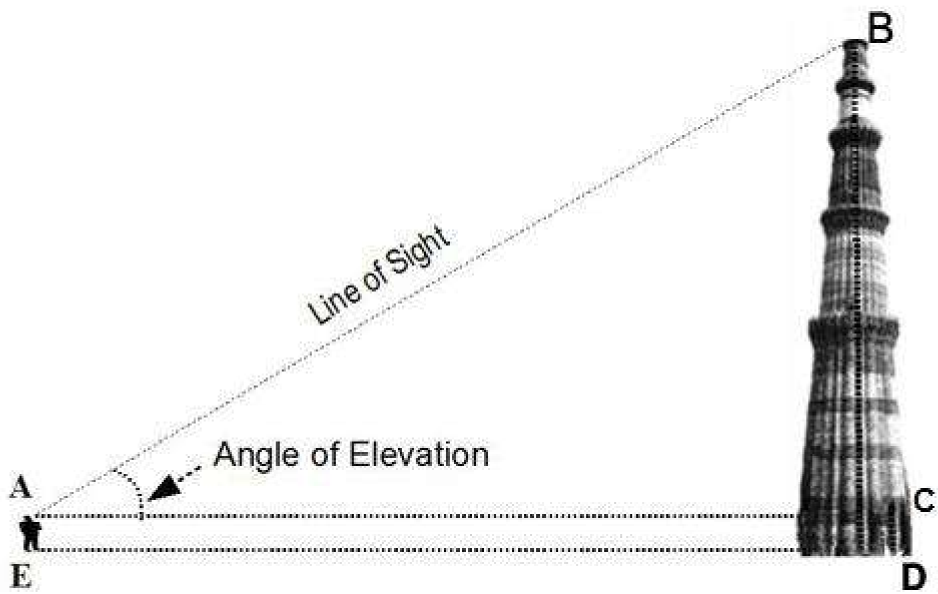


Fig 1

be imagined to be made, as shown in Fig 1.Can the student find out the height of the minar, without actually measuring it?

Example II

Suppose a girl is sitting on the balcony of her house located on the bank of a river .She is looking down at a flower pot on the stair of a temple situated nearby on the other bank of the river. A right triangle is

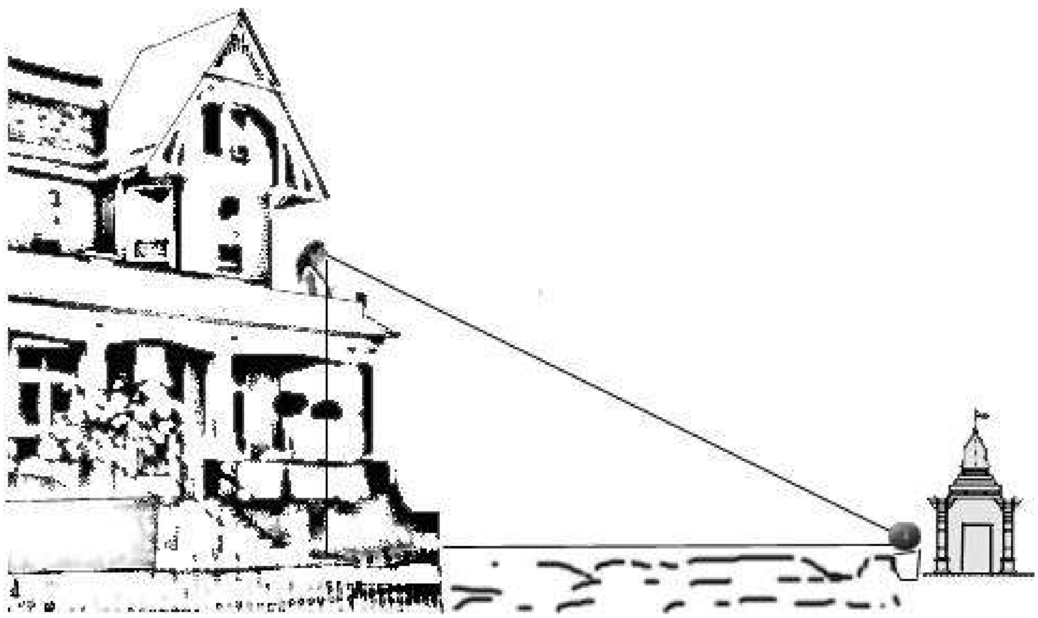


Fig 2

imagined to be made in this situation as shown in Fig.2. If you know the height at which the person is sitting. Can you find the width of the river? (Fig 2)

Example III:

Suppose a hot air balloon is flying in the air. A girl happens to spot the balloon in the sky and runs to her mother to tell her about it. Her mother rushes out of the house to look at the balloon. Now when the girl had

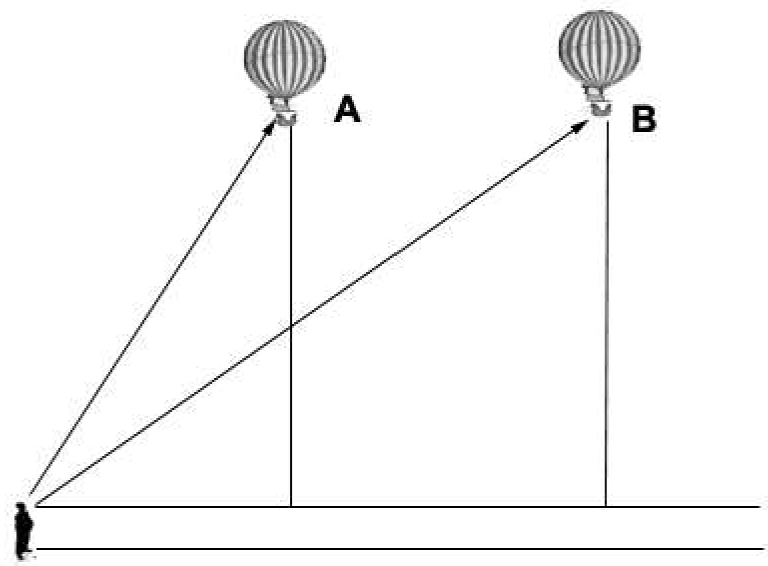


Fig 3

spotted the balloon initially it was at point A. when both the mother and daughter came out to see it. it had already travelled to another point B. Can you find the altitude of B from the ground? (Fig 3)

In all the situations given above, the **distance** or **height** can be found by using some mathematical techniques, which come under a branch of mathematics called **'trigonometry'**. The word 'trigonometry' is derived from the Greek words 'tri' (meaning three), 'gon' (meaning sides) and 'metron' (meaning measure).In fact, trigonometry is the study of relationship between the sides and angles of a triangle.

Trigonometry was invented because its need arose in astronomy. Since then the astronomers have used it, for instance , to calculate distance from the earth to the planets and stars. Trigonometry is also used in geography and in navigation also.

For determining height and distance of objects we make use of trigonometric ratios of an angle.

<u>What are trigonometric ratios of an Angle</u>?

Ratios of the sides of a right triangle w.r.t. its acute angles, are called **trigonometry ratios of the angle**.

Let ABC be a right angled triangle. Here, $\angle CAB$ is an acute angle(Fig 4). Let its measure be $\angle A$. The trigonometric ratios of the angle A in right triangle ABC are defined as follows:

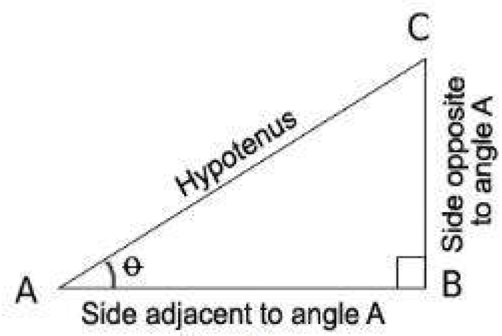


Fig 4

The trigonometric ratios of the angle A in right triangle ABC are defined as follows:

$$\text{Sine of } \angle A = \frac{\text{side opposite to angle A}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\text{Cosine of } \angle A = \frac{\text{side adjacent to angle A}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{Tangent of } \angle A = \frac{\text{side opposite to angle A}}{\text{side adjacent to angle A}} = \frac{BC}{AB}$$

Cosecant of $\angle A$, Secant of $\angle A$ and Cotangent of $\angle A$ are the reciprocals of Sine of $\angle A$, Cosine of $\angle A$ and tangent $\angle A$ repectively.

The ratios defined above are abbreviated as sin A, cos A, cosec A, sec A , tan A and cot A respectively. Also, observe that tan A is the quotient sin A/ cos A. So, the trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and the length of its sides.

Trigonometric Ratios of Some specific Angles

We shall now give the values of the trigonometric ratios for some specific angles, namely 0^0, 30^0, 45^0 ,60^0, and 90^0degrees.

Since our course is restricted to these angles, we are not giving values of trigonometric values for anther angles. However, we can get the values from the trigonometric tables.

Here is a ready reference for these values.

$\angle A$	0^0	30^0	45^0	60^0	90^0
sin A	0	½	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos A	1	$\sqrt{3}/2$	$1/\sqrt{2}$	½	0
tan A	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
sec A	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0

Here is an example which illustrates the use of the values given in the table above.

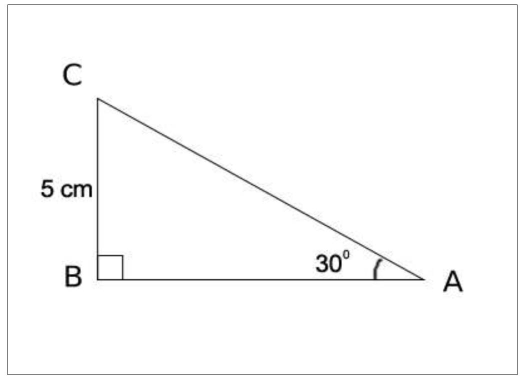
Problem I

In ΔABC, right_angled at B, $AB=5$ cm and $< ABC=30^0$.
Find the lenghths of BC and CA.

Solution

From the figure,

$$\frac{AB}{AC} = \text{Sin } 30^0 = \frac{1}{2}$$



» $AC=2\ AB= 2\times5\ \text{cm}=10\ \text{cm}$

Also, from the figure,

$$\frac{AB}{BC} = \tan 30^0 = \frac{1}{\sqrt{3}}$$

» $BC= \sqrt{3}\ AB$

» $=\sqrt{3}x5=5\sqrt{3}$ cm

Verification

You have the three sides of the right-angled triangle ABC as

$AB=5$ cm ; $AC=10$ cm ; $BC=5\sqrt{3}$ cm

Now

$$AB^2 + BC^2 =5^2+(5\sqrt{3})^2$$
$$=5^2+5^2\times3$$
$$=5^2\ (1+3) = 5^2\times4=100$$

Also,$AC^2 = 10^2 =100$ cm.

Hence, ΔABC is a right –angled triangle

Heights and Distances

We shall now see how Trigonometry is used for finding the height and distances of various objects, without actually measuring them.

Before going to problem, let us learn about to kinds of angles:

Angle of Elevation

In the figure, the line AB drawn from the eye of the student top of the Minar is called the line of sight . The student is looking at the top of minar . The angle BAC ,so formed by the line of sight of the horizontal, is called the **angle of elevation** of the top of the minar from eye of the Student.

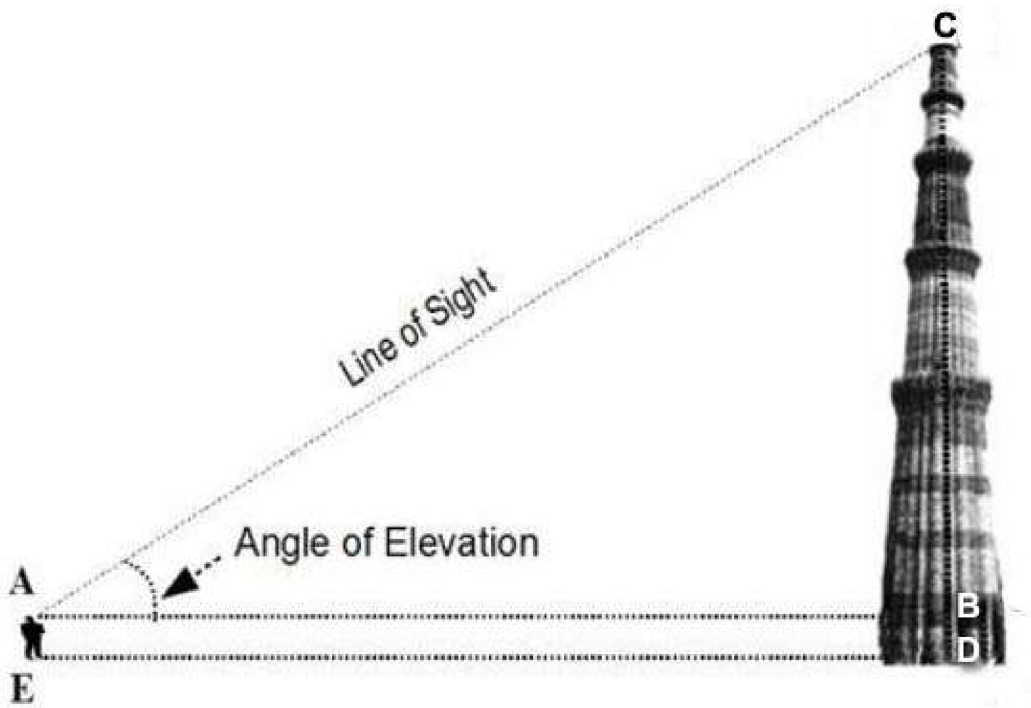


Fig 6

Now, consider the situation given in the beginning. The girl sitting on the balcony is looking down at a flowerpot placed on a stair of the temple. In this case, the line of sight is below the horizontal is called the **angle of depression.**

Angle of depression.

Now, you may identify the lines of sight, and the angles so formed in the above figure are the angle of elevation or angles of depression?(Fig 7)

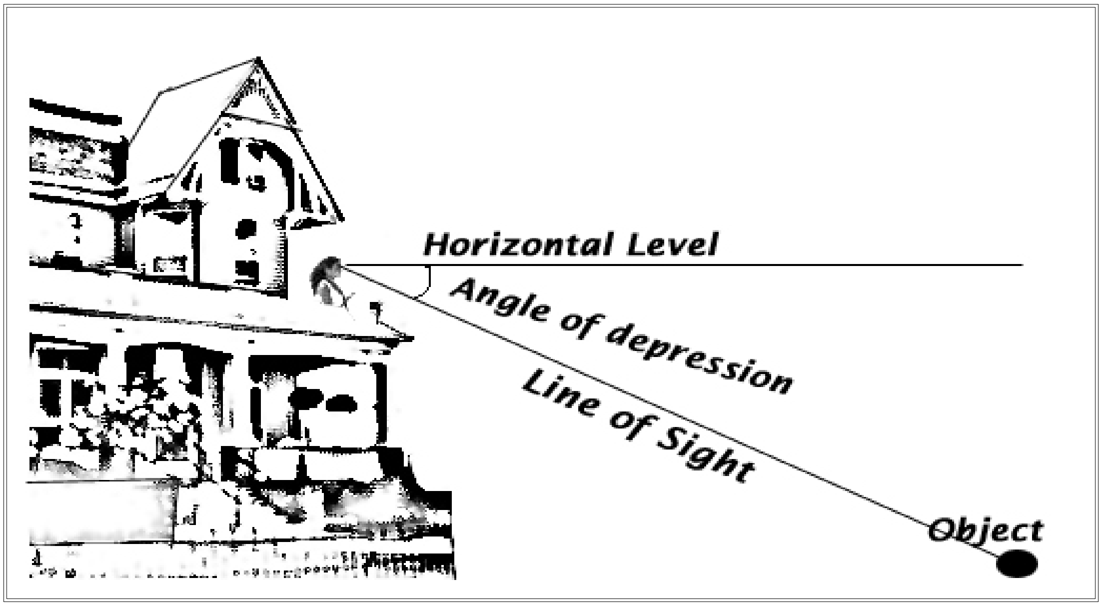


Fig 7

Problem II
Finding the Height of Qutub minar

Let us refer to Fig. 6 (Fig 8) again. If you want to find the height BD of the minar without actually measuring it, what information do you need? You would need to know the following:

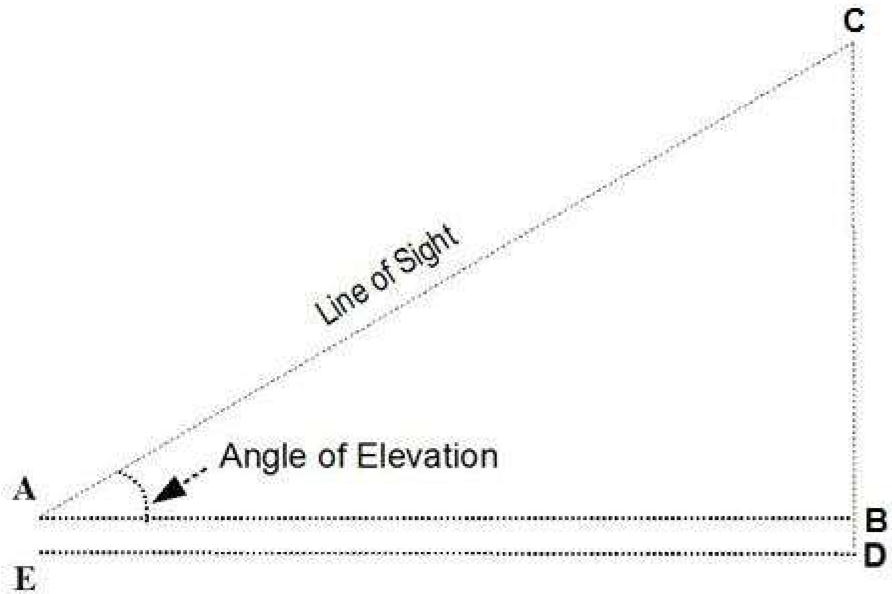


Fig 8

(i) The distance DE at which the student is standing from the foot of the minar. Let it be 40 meters
(ii) The angle of elevation, $\angle BAC$, of the top of the minar. Let it be 60^0
(iii) The height AE of the student. (Let it be 105cm)

From ΔBAC,

$$\frac{BC}{AC} = \frac{BC}{DE} = \tan 60^0$$

» $BC = 40 \times \sqrt{3}$ m

Hence, BD $= BC + CD$

$= BC + AE$

$= \left(40\sqrt{3} + \frac{105}{100}\right)$ m

$= (69.28 + 1.05)$ m $= 70.33$ m

Qutb-Minar in red and buff standstone is the highest tower in India. It has a diameter of 14.32 m at the base and about 2.75 m on the top with a height of 72.5 m.

- ***Archaeological Survey of India, Government of India***