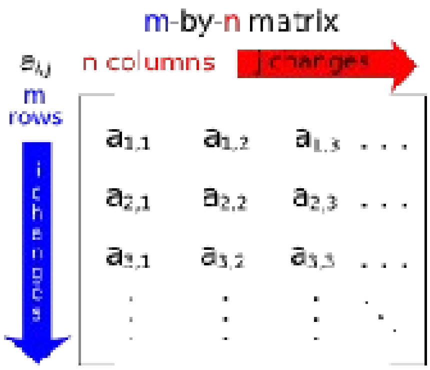
Introduction to Matrix

In mathematics, a matrix (plural matrices) is a rectangular table of elements (or entries), which may be numbers or, more generally, any abstract quantities that can be added and multiplied. Matrices are used to describe linear equations, keep track of the coefficients of linear transformations and to record data that depend on multiple parameters.

Matrices are described by the field of matrix theory. They can be added, multiplied, and decomposed in various ways, which also makes them a key concept in the field of linear algebra.



Example

$\mathbf{A} = \begin{bmatrix} 9 & 8 & 6 \\ 1 & 2 & 7 \\ 4 & 9 & 2 \\ 6 & 0 & 5 \end{bmatrix}$	or	$\mathbf{A} = \begin{pmatrix} 9 & 8 & 6 \\ 1 & 2 & 7 \\ 4 & 9 & 2 \\ 6 & 0 & 5 \end{pmatrix}$

The matrix is a 4 x 3 matrix. The element a2,3 or a [2,3] is 7. In terms of the mathematical definition given above, this matrix is a function A: {1,2,3,4} x {1,2,3} -> R and, for example, a ((2,3)) = 7 and A ((3,1)) = 4.

The matrix

R = [1 2 3 4 5 6 7 8 9]

is a 1 x 9 matrix, or 9-element row vector.

Basic operations

Matrix Addition

Two or more matrices of identical dimensions m and n can be added. Given m-by-n matrices A and B, their sum A+B is the m-by-n matrix computed by adding corresponding elements:

$$\mathbf{A}+\mathbf{B} = (a_{i,j})_{1\le i\le m;\,1\le j\le n} + (b_{i,j})_{1\le i\le m;\,1\le j\le n} = (a_{i,j}+b_{i,j})_{1\le i\le m;\,1\le j\le n}.$$

For example:

$$\begin{bmatrix}1 & 3 & 1\\ 1 & 0 & 0\\ 1 & 2 & 2\end{bmatrix} + \begin{bmatrix}0 & 0 & 5\\ 7 & 5 & 0\\ 2 & 1 & 1\end{bmatrix} = \begin{bmatrix}1+0 & 3+0 & 1+5\\ 1+7 & 0+5 & 0+0\\ 1+2 & 2+1 & 2+1\end{bmatrix} = \begin{bmatrix}1 & 3 & 6\\ 8 & 5 & 0\\ 3 & 3 & 3\end{bmatrix}.$$

Another, much less often used notion of matrix addition is the direct sum.

Matrix Substraction

If A and B are matrices of the same type then the subtraction is found by subtracting the corresponding elements aij-bij

Here is an example of subtracting matrices

$$A-B=\begin{pmatrix}1 & 2 & 3\\ 1 & 0 & 2\end{pmatrix}-\begin{pmatrix}2 & 1 & 2\\ 1 & 0 & 3\end{pmatrix}=\begin{pmatrix}-1 & 1 & 1\\ 0 & 0 & -1\end{pmatrix}$$

Matrix multiplication

Multiplication of two matrices is well-defined only if the number of columns of the left matrix is the same as the number of rows of the right matrix. If A is an m-by-n matrix and B is an n-by-p matrix, then their matrix product AB is the m-by-p matrix given by:

for each pair (i,j).

For example:

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} (1 \times 3 + 0 \times 2 + 2 \times 1) & (1 \times 1 + 0 \times 1 + 2 \times 0) \\ (-1 \times 3 + 3 \times 2 - 1 \times 1) & (-1 \times 1 - 3 \times 1 - 1 \times 0) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}.$$

Matrix multiplication has the following properties:

- (AB)C = A(BC) for all k-by-m matrices A, m-by-n matrices B and n-by-p matrices C ("associativity").
- (A+B)C = AC+BC for all m-by-n matrices A and B and n-by-k matrices C ("right distributivity").
- C(A+B) = CA+CB for all m-by-n matrices A and B and k-by-m matrices C ("left distributivity").

Matrix multiplication is not commutative; that is, given matrices A and B and their product defined, then generally AB BA. It may also happen that AB is defined but BA is not defined.